Supplementary Problems

2.31 Use the equivalent systems method to derive the differential equation governing the motion of the system of Fig. 2-22. Use x as the generalized coordinate. Determine the system's natural frequency.

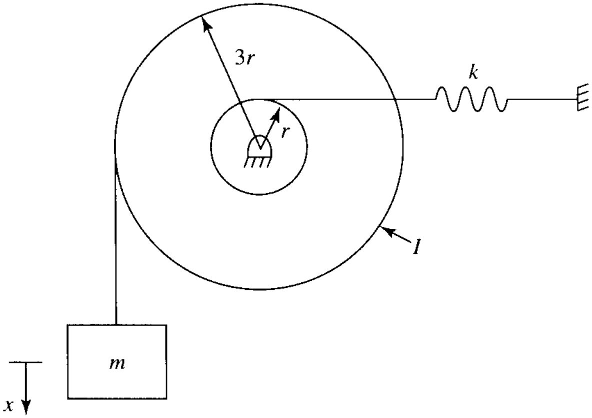

Fig. 2-22

Ans.

$$\left(m+\frac{I}{9r^2}\right)\ddot{x}+\frac{k}{9}x=0, \qquad \omega_n=\sqrt{\frac{kr^2}{I+9mr^2}}$$

2.32 Use the equivalent system method to derive the differential equation governing the motion of the system of Fig. 2-23. Use θ as the generalized coordinate assuming small θ. Determine the system's natural frequency.

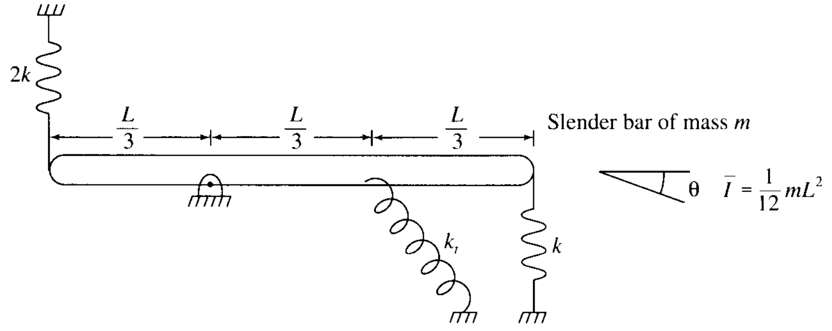

Fig. 2-23

Ans.

$$\frac{1}{9}mL^2\ddot{\theta}+\left(\frac{2}{3}kL^2+k_t\right)\theta=0, \qquad \omega_n=\sqrt{\frac{6kL^2+9k_t}{mL^2}}$$

2.33 Use the equivalent system method to derive the differential equation governing the motion of the system of Fig. 2-24. Use x as the generalized coordinate. Assume small x and determine the system's natural frequency.

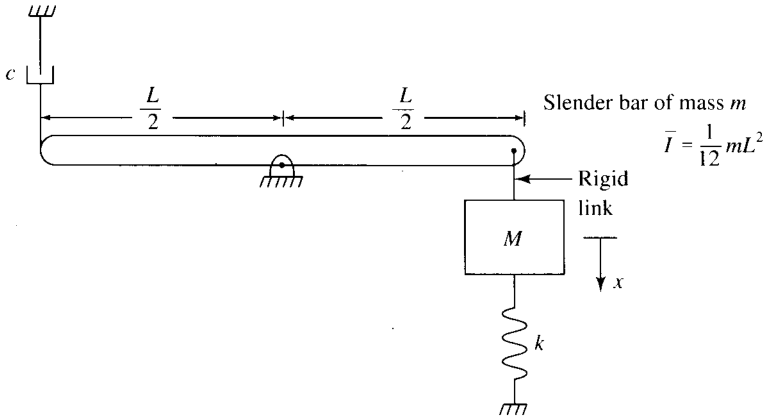

Fig. 2-24

Ans.

$$\left(\frac{1}{3}m+M\right)\ddot{x}+c\dot{x}+kx=0, \qquad \omega_n=\sqrt{\frac{3k}{m+3M}}$$

2.34 Use the free body diagram method to derive the differential equation governing the motion of the system of Fig. 2-25. Use θ as the generalized coordinate, assuming small θ. Assume the structure is composed of two slender rods welded together.

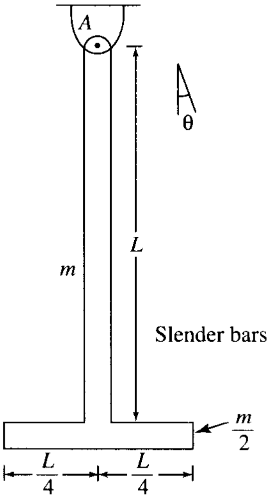

Fig. 2-25

Ans.

$$\frac{81}{96}mL^2\ddot{\theta} + mgL\theta = 0$$

2.35 Use the free body diagram method to derive the differential equation governing the motion of the system of Fig. 2-26. Use θ as the generalized coordinate, assuming small θ.

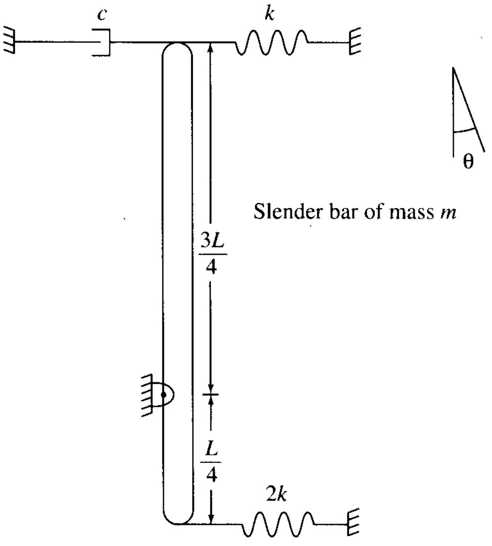

Fig. 2-26

Ans.

$$\frac{7}{48}mL^2\ddot{\theta} + \frac{9}{16}cL^2\dot{\theta} + \left(\frac{11}{16}kL^2 - mg\frac{L}{4}\right)\theta = 0$$

2.36 A 300-kg block is attached to four identical springs, each of stiffness 2.3×10^5 N/m, placed in parallel. Determine the system's natural frequency in hertz.

Ans. 8.81 Hz

2.37 A thin disk of mass moment of inertia 5.8 kg-m^2 is attached to the end of a 2.5-m aluminum ($G = 40 \times 10^9$ N/m^2) shaft of 10 cm diameter. What is the natural frequency of torsional oscillation of the disk?

Ans. 164.6 rad/s

2.38 Determine the natural frequency of the system of Fig. 2-27.

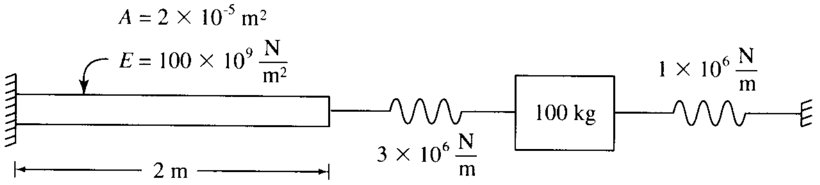

Fig. 2-27

Ans. 132.3 rad/s

2.39 When empty the static deflection of a 2000-lb vehicle is 0.8 in. What is the vehicle's natural frequency when it is carrying a 200-lb passenger and 250 lb of cargo?

Ans. 19.9 rad/s

2.40 The location of the center of mass and the mass moment of inertia of the connecting rod of Fig. 2-28 are unknown. When the rod is pinned at A, its natural frequency is observed as 20 rad/s. When a 250-g mass is added to the free end, the system's natural frequency is observed as 10 rad/s. Determine the location of the center of mass ℓ.

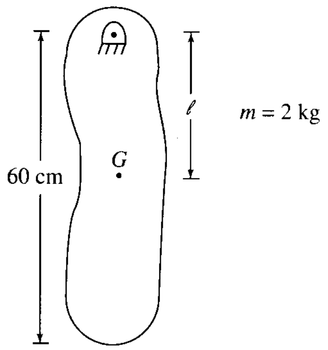

Fig. 2-28

Ans. 0.512 m

2.41 A rotor of mass moment of inertia 2.5 kg-m^2 is to be attached at the end of a 60-cm circular steel ($G = 80 \times 10^9$ N/m^2) shaft. What is the range of shaft diameters such that the torsional natural frequency of the system is between 100 and 200 Hz?

Ans. $9.32 \text{ cm} < D < 13.2 \text{ cm}$

2.42 A uniform 45-kg flywheel of inner radius 80 cm and outer radius 100 cm is swung as a pendulum about a knife edge support on its inner rim. Its period is observed as 2.1 s. Determine the flywheel's centroidal moment of inertia.

Ans. $I = 10.65\ \text{kg-m}^2$

2.43 A particle of mass m is attached to the midpoint of a taut string of length L and tension T, as shown in Fig. 2-29. Determine the particle's natural frequency of vertical vibration.

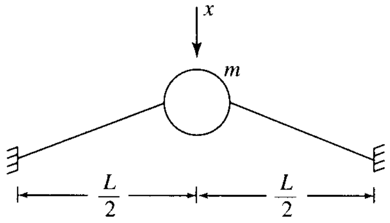

Fig. 2-29

Ans.

$$\omega_n = \sqrt{\frac{4T}{mL}}$$

2.44 The disk in the system of Fig. 2-30 rolls without slip. Determine the value of c such that the system has a damping ratio of 0.2.

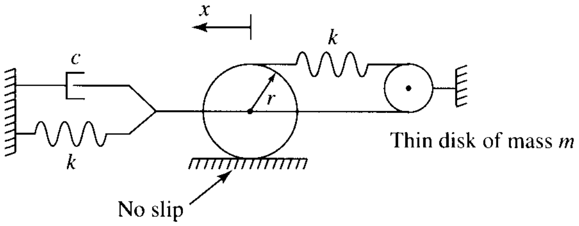

Fig. 2-30

Ans.

$$c = 1.55\sqrt{mk}$$

2.45 What is the value of c such that the system of Fig. 2-31 is critically damped if $m = 20$ kg and $k = 10{,}000$ N/m?

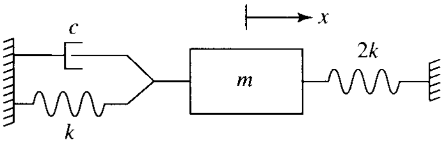

Fig. 2-31

Ans. $c = 1.55 \times 10^3$ N-s/m

2.46 A 200-kg block is attached to a spring of stiffness 50,000 N/m in parallel with a viscous damper. The period of free vibration of this system is observed as 0.417 s. What is the value of the damping coefficient?

Ans. $c = 1.91 \times 10^3$ N-s/m

2.47 For the recoil mechanism designed in the solution of Problem 2.17, what is the initial velocity of the recoil mechanism that leads to a recoil of 5 cm?

Ans. 1.79 m/s

2.48 What is the minimum damping ratio for an underdamped system such that its overshoot is limited to 10 percent.

Ans. $\zeta = 0.591$

2.49 A 1000-kg machine is placed on a vibration isolator of stiffness 1×10^6 N/m. The machine is given an initial displacement of 5 cm and released. After 10 cycles the machine's amplitude is 1 cm. What is the damping ratio of the system?

Ans. $\zeta = 0.026$

2.50 A 100-kg block is attached to a spring of stiffness 1.5×10^6 N/m in parallel with a viscous damper of damping coefficient 4900 N-s/m. The block is given an initial velocity of 5 m/s. What is its maximum displacement?

Ans. 30.9 mm

2.51 Solve Problem 2.50 if $c = 29{,}000$ N-s/m.

Ans. 13.4 mm

2.52 How long after being given the initial velocity will it take the system of Problem 2.51 to return permanently to within 1 mm of equilibrium.

Ans. 0.0515 s

2.53 The slender bar in the system of Fig. 2-32 is rotated 5° from equilibrium and released. Determine the time dependent response of the system if $m = 2$ kg, $L = 80$ cm, $r = 10$ cm, $k = 20{,}000$ N/m, and $c = 300$ N-s/m.

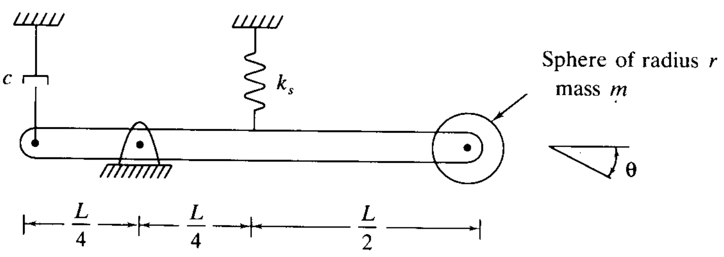

Fig. 2-32

Ans.

$$\theta(t) = 0.0895e^{-6.61t} \sin(28.9t + 1.35)$$

2.54 Solve Problem 2.53 if $c = 1500$ N-s/m.

Ans.

$$\theta(t) = 0.144e^{-18.6t} - 0.055e^{-47.6t}$$

2.55 A spring-dashpot mechanism is designed such that a system is critically damped when the system has a mass m. What is the damping ratio of a system using this mechanism with a mass (*a*) $3m/4$, (*b*) $4m/3$?

Ans. (a) 1.15, (b) 0.866

2.56 If the initial conditions for the motion of a critically damped system are of opposite sign, overshoot is possible. Derive a relationship that the initial conditions x_0 and $\dot{x}_0$ must satisfy in order for overshoot to occur.

Ans.

$$\frac{x_0}{\dot{x}_0 + \omega_n x_0} < 0$$

2.57 A 35-kg block is connected to a spring of stiffness 1.7×10^5 N/m. The coefficient of friction between the block and the surface on which it slides is 0.11. The block is displaced 10 mm from equilibrium and released. (*a*) What is the amplitude of motion at the end of the first cycle? (*b*) How many cycles of motion occur?

Ans. (*a*) 9.11 mm, (*b*) 11

2.58 A 50-kg block is attached to a spring of stiffness 200,000 N/m and slides on a surface that makes an angle of 34° with the horizontal. For what values of μ, the coefficient of friction between the block and the surface, will motion cease during the 10th cycle when the block is displaced 1 cm from equilibrium and released?

Ans. $0.120 < \mu < 0.133$